Centre Number		Candidate Number	
Surname	MR BARTON'S		
Other Names	SOLUTIONS		
Candidate Signature			

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2015

Mathematics MFP1

Unit Further Pure 1

Friday 5 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The quadratic equation $2x^2 + 6x + 7 = 0$ has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

[2 marks]

(b) Find a quadratic equation, with integer coefficients, which has roots $\alpha^2 - 1$ and $\beta^2 - 1$.

[5 marks]

(c) **Hence** find the values of α^2 and β^2.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

1 a) $\alpha + \beta = -\frac{6}{2} = -3$

$\alpha\beta = \frac{7}{2} = 3.5$

b) $\boxed{\text{SUM}}$ $\alpha^2 - 1 + \beta^2 - 1$

$= \alpha^2 + \beta^2 - 2$

$= (\alpha + \beta)^2 - 2\alpha\beta - 2$

$= (-3)^2 - 2(3.5) - 2 = 0$

$\boxed{\text{PRODUCT}}$ $(\alpha^2 - 1)(\beta^2 - 1)$ * From ~~prat~~ $\boxed{\text{SUM}}$

$= \alpha^2\beta^2 - \alpha^2 - \beta^2 + 1$

$= \alpha^2\beta^2 - (\alpha^2 + \beta^2) + 1$

$= (\alpha\beta)^2 - 2^* + 1$

$3.5^2 - 2 + 1 = 11.25$

$x^2 - \boxed{\text{SUM}}\,x + \boxed{\text{PRODUCT}} = 0$

$\rightarrow x^2 - 0x + 11.25 = 0$

$\rightarrow 4x^2 + 45 = 0$

QUESTION PART REFERENCE	Answer space for question 1

c) SOLVE: $4x^2 + 45 = 0$

$\rightarrow 4x^2 = -45$

$\rightarrow x^2 = -45/4$

$\rightarrow x = \pm\sqrt{-45/4}$

$\rightarrow x = \pm i\sqrt{45/4}$

$\rightarrow$ ROOTS are $\pm i\sqrt{45/4}$

$\rightarrow (\alpha^2 - 1)$ & $(\beta^2 - 1) = \pm i\sqrt{45/4}$

$\rightarrow \alpha^2$ & $\beta^2 = 1 \pm i\sqrt{45/4}$

Turn over ▶

2 (a) Explain why $\int_0^4 \frac{x-4}{x^{1.5}}\,dx$ is an improper integral.

[1 mark]

(b) Either find the value of the integral $\int_0^4 \frac{x-4}{x^{1.5}}\,dx$ or explain why it does not have a finite value.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

2 a) when $x = 0$, the integral is undefined as you are dividing by $\sqrt{x^3}$ which $= 0$

b) $\int_0^4 \frac{x-4}{x^{1.5}}\,dx = \int x^{-0.5} - 4x^{-1.5}\,dx$

$= \left[\frac{x^{0.5}}{0.5} - \frac{4x^{-0.5}}{-0.5} \right]_0^4$

Change 0 to K

$\rightarrow \left[2\sqrt{x} + 8/\sqrt{x} \right]_K^4$

$\rightarrow \left(2\sqrt{4} + 8/\sqrt{4}\right) - \left(2\sqrt{K} + 8/\sqrt{K}\right)$

As $K \rightarrow 0$, $8/\sqrt{K}$ does not $\rightarrow$ a limit

as $K \rightarrow 0$, $8/\sqrt{K} \rightarrow \infty$

$\therefore$ Integral does NOT have a finite limit

3 (a) Show that $(2+\mathrm{i})^3$ can be expressed in the form $2+b\mathrm{i}$, where b is an integer.

[3 marks]

(b) It is given that $2+\mathrm{i}$ is a root of the equation

$$z^3+pz+q=0$$

where p and q are real numbers.

(i) Show that $p=-11$ and find the value of q.

[4 marks]

(ii) Given that $2-\mathrm{i}$ is also a root of $z^3+pz+q=0$, find a quadratic factor of z^3+pz+q with real coefficients.

[2 marks]

(iii) Find the real root of the equation $z^3+pz+q=0$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 3

3 a) $(2+i)^3 = 2^3 + 3(2)^2 i + 3(2)(i)^2 + i^3$

$= 8 + 12i - 6 - i$

$= 2 + 11i$

b) $(2+i)$ is a root

$\rightarrow (2+i)^3 + p(2+i) + q = 0$

$2+11i + 2p + ip + q = 0$

$\boxed{\text{REAL}}$ $2 + 2p + q = 0$

$\boxed{\text{IMAG}}$ $11i + ip = 0 \rightarrow p = -11$

REAL: $2 + 2(-11) + q = 0$

$2 - 22 + q = 0 \rightarrow q = 20$

QUESTION PART REFERENCE

Answer space for question 3

ii) Roots must be $[z-(2+i)]$ and $[z-(2-i)]$

So, factor must be $[z-(2+i)][z-(2-i)]$

$$= z^2 + 4 - i^2 - 4z$$
$$= z^2 - 4z + 5$$

iii) $(z^2 - 4z + 5)(z+4) = z^3 - 11z + 20$

Factor

$\Rightarrow$ root is -4

Turn over ▶

4 (a) Find the general solution, in degrees, of the equation

$$2\sin(3x + 45°) = 1$$

[5 marks]

(b) Use your general solution to find the solution of $2\sin(3x + 45°) = 1$ that is closest to $200°$.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 4

4 a) $\sin(3x + 45) = 0.5$

Key angle : $\sin^{-1}(0.5) = 30°$

$\theta = 360n + a, \quad \theta = 360n + (180 - a)$

$\Rightarrow 3x + 45 = 360n + 30, \quad 3x + 45 = 360n + 150$

$\Rightarrow 3x = 360n - 15, \quad 3x = 360n + 105$

$\Rightarrow x = 120n - 5, \quad x = 120n + 35$

b) Try $n = 2$

$\Rightarrow x = 240 - 5 = 235$ ← closest!

Try $n = 1$

$\Rightarrow x = 120 + 35 = 155$

5 (a) The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} -2 & c \\ d & 3 \end{bmatrix}$.

Given that the image of the point $(5, 2)$ under the transformation represented by $\mathbf{A}$ is $(-2, 1)$, find the value of c and the value of d.

[4 marks]

(b) The matrix $\mathbf{B}$ is defined by $\mathbf{B} = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}$.

(i) Show that $\mathbf{B}^4 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix.

[2 marks]

(ii) Describe the transformation represented by the matrix $\mathbf{B}$ as a combination of two geometrical transformations.

[5 marks]

(iii) Find the matrix $\mathbf{B}^{17}$.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 5**
a)	i) $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$
	$\begin{pmatrix} -2 & c \\ d & 3 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$
	$5(-2) + 2c = -2$
	$\rightarrow -10 + 2c = -2$
	$\rightarrow 2c = 8 \rightarrow c = 4$
	$5d + 6 = 1$
	$5d = -5 \rightarrow d = -1$

QUESTION PART REFERENCE

Answer space for question 5

ii) Find B^2:

$$\begin{pmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{pmatrix} \quad \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$$

B^4:

$$\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} \quad \begin{pmatrix} -16 & 0 \\ 0 & -16 \end{pmatrix}$$

$$\therefore B^4 = \begin{pmatrix} -16 & 0 \\ 0 & -16 \end{pmatrix}$$

$$= -16 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -16 I$$

ii) $\begin{pmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{pmatrix}$ = Enlargement SF 2, as B^4 = Enlargement SF4

$$= 2 \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Turn over ▶

QUESTION PART REFERENCE

Answer space for question 5

$$= 2 \begin{pmatrix} \cos(-45) & -\sin(-45) \\ \sin(-45) & \cos(-45) \end{pmatrix}$$

= Enlargement scale factor 2

AND Rotation clockwise 45° about $(0,0)$

iii) $B^{17} = \left[B^4\right]^4 \times B$

$$= \left[-16I\right]^4 \times \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{pmatrix}$$

$$= 65536 \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}$$

6 A curve C_1 has equation

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

(a) Sketch the curve C_1, stating the values of its intercepts with the coordinate axes.

[2 marks]

(b) The curve C_1 is translated by the vector $\begin{bmatrix} k \\ 0 \end{bmatrix}$, where $k < 0$, to give a curve C_2.

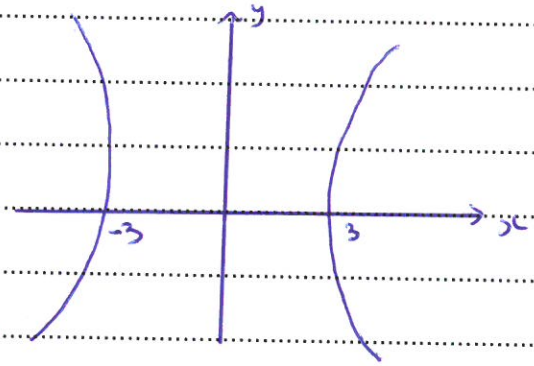
Given that C_2 passes through the origin $(0, 0)$, find the equations of the asymptotes of C_2.

[3 marks]

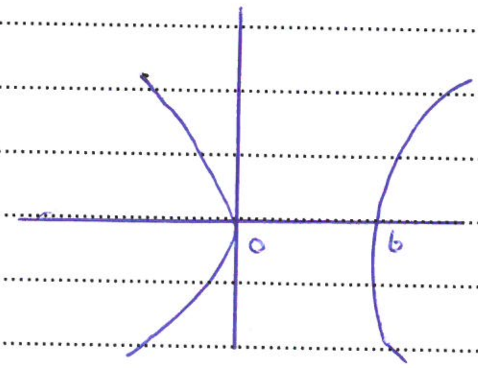
QUESTION PART REFERENCE	Answer space for question 6

6 a) crosses x-axis at $\frac{x^2}{9} - 0 = 1$

$\rightarrow x = 3, -3$



b) $K < 0 \rightarrow K = -3$



Asymptotes of original

$\Rightarrow \frac{x}{3} = \pm \frac{y}{4}$

New Asymptotes

$\rightarrow \frac{x+3}{3} = \pm \frac{y}{4}$

7 (a) The equation $2x^3 + 5x^2 + 3x - 132\,000 = 0$ has exactly one real root α.

(i) Show that α lies in the interval $39 < \alpha < 40$.

[2 marks]

(ii) Taking $x_1 = 40$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α. Give your answer to two decimal places.

[3 marks]

(b) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n} 2r(3r+2) = n(n+p)(2n+q)$$

where p and q are integers.

[5 marks]

(c) (i) Express $\log_8 4^r$ in the form λr, where λ is a rational number.

[1 mark]

(ii) By first finding a suitable cubic inequality for k, find the greatest value of k for which $\sum_{r=k+1}^{60} (3r+2)\log_8 4^r$ is greater than $106\,060$.

[4 marks]

QUESTION PART REFERENCE | **Answer space for question 7**

7 a) i) $f(x) = 2x^3 + 5x^2 + 3x - 132\,000$

$f(39) = 2(39)^3 + 5(39)^2 + 3(39) - 132\,000 = -5640$

$f(40) = 2(40)^3 + 5(40)^2 + 3(40) - 132\,000 = 4120$

Change of sign, $\therefore$ $39 < \alpha < 40$

ii) $x_1 = 40$

$f(x_1) = f(40) = 4120$

$f'(x) = 6x^2 + 10x + 3$

$f'(x_1) = f'(40) = 6(40)^2 + 10(40) + 3$

$= 10{,}003$

QUESTION PART REFERENCE

Answer space for question 7

$\rightarrow \quad x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 40 - \dfrac{4120}{10{,}003}$

$= 39.58812..$

$= 39.59 \quad (2dp)$

b) $\displaystyle\sum_{1}^{n} 2r(3r+2) = 6\sum r^2 + 4\sum r$

$= 6\left[\frac{n}{6}(n+1)(2n+1)\right] + 4\left[\frac{n}{2}(n+1)\right]$

$= n(n+1)(2n+1) + 2n(n+1)$

$= n(n+1)\left[2n+1+2\right]$

$= n(n+1)(2n+3)$

c) i) $\log_8(4^r) \Rightarrow \frac{2}{3}r$

as $8^{\frac{2}{3}} = 4$

QUESTION PART REFERENCE	Answer space for question 7
	c) ii) $\sum_{k+1}^{60} (3r+2)\log_8 4^r$
	$= \sum_{k+1}^{60} (3r+2)\frac{2}{3}r \quad = \frac{1}{3}[\text{Part b)}]$
	$= \sum_{k+1}^{60} 2r^2 + \frac{4}{3}r$
	$\sum_{k+1}^{60} = \sum_{1}^{60} - \sum_{1}^{k}$
	using (b) : $\sum_{1}^{60} = \frac{1}{3}[n(n+1)(2n+3)]$
	$= \frac{1}{3}[60 \times 61 \times 123]$
	$= 150{,}060$
	Need greatest integer such that:
	$150{,}060 - \frac{1}{3}[k(k+1)(2k+3)] > 106{,}066$
	$\rightarrow \frac{k}{3}(k+1)(2k+3) < 44{,}000$
	$\rightarrow k(k+1)(2k+3) < 132\,000$
	$\rightarrow 2k^3 + 5k^2 + 3k - 132\,000 < 0$
	Try $k = 38$ ✗
	$k = 39$ ✓ $\rightarrow k = 39$

8 A curve C has equation

$$y = \frac{x(x-3)}{x^2+3}$$

(a) State the equation of the asymptote of C.

[1 mark]

(b) The line $y = k$ intersects the curve C. Show that $4k^2 - 4k - 3 \leqslant 0$.

[5 marks]

(c) **Hence** find the coordinates of the stationary points of the curve C.

(No credit will be given for solutions based on differentiation.)

[5 marks]

QUESTION PART REFERENCE

Answer space for question 8

8 a) As $x \to \infty$, $y \to \frac{1}{1} \to \boxed{y = 1}$

b) $k = \dfrac{x(x-3)}{x^2+3}$

$\to k(x^2+3) = x(x-3)$

$kx^2 + 3k = x^2 - 3x$

$kx^2 - x^2 + 3x + 3k = 0$

$(k-1)x^2 + 3x + 3k = 0$

As line intersects curve, roots are real,

so $b^2 - 4ac \geq 0$

$\to 3^2 - 4(k-1)(3k) \geq 0$

$9 - 12k^2 + 12k \geq 0$

$12k^2 - 12k - 9 \leq 0$

$4k^2 - 4k - 3 \leq 0$

QUESTION PART REFERENCE	Answer space for question 8

c) At stationary point, $b^2 - 4ac = 0$

$\rightarrow 4k^2 - 4k - 3 = 0$

$(2k+1)(2k-3) = 0$

$k = -0.5$ $\quad$ $k = 1.5$

$\rightarrow y = -0.5$ $\quad$ $\rightarrow y = 1.5$

Find x using $(k-1)x^2 + 3x + 3k = 0$

~~-0.5~~ $\boxed{k = -0.5}$ $\quad$ $\boxed{k = 1.5}$

$-1.5x^2 + 3x - 1.5 = 0$ $\quad$ $0.5x^2 + 3x + 4.5 = 0$

$\rightarrow x^2 - 2x + 1 = 0$ $\quad$ $\rightarrow x^2 + 6x + 9 = 0$

$(x-1)(x-1) = 0$ $\quad$ $(x+3)(x+3) = 0$

$\rightarrow x = 1$ $\quad$ $\rightarrow x = -3$

$(1, -0.5)$ $\quad$ $(-3, 1.5)$

Stationary points